Hamilton-Jacobi equations with discontinuous coefficients

References for the spring school in Besançon, 2025

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Abstract

We give a few key references for the lectures. The literature about Hamilton-Jacobi equations with discontinuous is now so rich that it is not possible to give an exhaustive list of references. For this reason, the reader is warmly invited to review references contained in the below mentioned works.

- The convex case
  - 1D junction [6, 17, 18, 1, 13, 11]
  - Regional control [3, 4, 2]
  - Multi-dimensional junctions [12]
  - With second derivatives / vanishing viscosity limit [14]
- The non-convex case [10, 15, 16, 8]
- Comparison principles in the non-convex case [10, 15, 16, 7, 9]
- A survey book [5]

References

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